How can I find out whether Google Mail or Calendar are having any problems?

Google has an "App Status" dashboard that lists the current status of all of their services. It can be found here: Google App Status.